Estimating Disclosure Models

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Introduction

- Some of the most relevant information that investors receive is voluntarily disclosed by firms (e.g. management forecasts. See Beyer et al 2011)
- Regulators are concerned about managers withholding information from investors
- There is a reason: if the market is opaque, resources can't be allocated efficiently
- This creates adverse selection and causes allocation and wealth distribution effects
- Quantify the extent of this problem is important, but it's hard to do without theory

The Dye (1985) Model

- Ex-ante investors believe the firm value is $x \sim F(x)$
- A manager/owner has private information about x with probability $1 - p$. With probability p he does not have any private information and shares the belief, $x \sim F(x)$
- Let us denote $\theta \in \{0, 1\}$ the random variable describing whether the manager is informed or not
- The manager may disclose his information to the market (if any) or withhold it, but he can't lie about it.The manager can't prove he is uninformed.
- Based on the disclosure choice $d \in \{x, ND\}$ the market rationally prices the firm as

$$P(d) = E[x|d].$$

- (more generally, $P(d) = \alpha + \beta E[x|d]$)

Model Solution

- In equilibrium, only the managers who observe $x \geq \tau$ disclosure their signal, that is $d = x$, otherwise, they conceal it, hence $d = ND$, and receive the price of non disclosure P^{ND}.
- The equilibrium has a threshold structure
- When $x = \tau$, the manager is indifferent hence $P^{ND} = \tau$.
- The market rationally prices non-disclosing firms, hence

$$
\begin{aligned}
P^{ND} &= \underbrace{\frac{pE(x)}{p + (1-p)F(\tau)}}_{=0} + \frac{(1-p)F(\tau)E(x|x<\tau)}{p + (1-p)F(\tau)}. \\
\tau &= \frac{(1-p)F(\tau)E(x|x<\tau)}{p + (1-p)F(\tau)}
\end{aligned}
$$

- One can show there is a unique solution, τ, to the above equation (see Jung and Kwon 1988).

Empirical Implementation

- There are two important predictions of this model:
  - Disclosure is infrequent. That is $\Pr(\text{disclosure}) < 1$.
  - Managers disclose good news, $E(x|\text{disclosure}) > E(x)$
- In other words, managers tend to conceal "bad news"
- But how much information do managers conceal?

Empirical Implementation

- To answer this question, let's specialize the model with an ancillary assumption
- Assume that

$$x \sim N\left(\mu_x, \sigma_x\right).$$

- One can demonstrate (see BMM) that no matter what μ_x and σ_x are the probability of disclosure is equal to

$$\Pr\left(d = ND\right) = p + (1 - p)\,\Phi\left(\tau\right) \tag{1}$$

where τ solves the following non-linear equation:

$$\tau + \frac{(1 - p)\,\phi\left(\tau\right)}{p + (1 - p)\,\Phi\left(\tau\right)} = 0 \tag{2}$$

- where ϕ and Φ are the pdf and cdf of the Gaussian distribution.

```
function output=gtau(p)
%function that returns tau for a given prob. of uninf. p
% speed increased with polynomial approximation
F=@(x) normcdf(x);
f=@(x) normpdf(x);
output=fzero(@(z) (z+(((1-p)*f(z))/(p+(1-p)*F(z)))),-.2);
```

Estimating Strategic Withholding

- The probability of no disclosure is given by $\Gamma(p) \equiv p + (1-p)\Phi(\tau(p))$
- This is useful empirically. If you have an estimate of $\Pr(d = ND)$ (the frequency of non-disclosure periods) you may estimate the probability of being uninformed p as

$$\hat{p} = \Gamma^{-1}\left(\Pr\widehat{(d = ND)}\right).$$

```
function output=phat(x)
%function that returns estimated p given frequency of non disc
F=@(x) normcdf(x);
f=@(x) normpdf(x);
output=fzero(@(p) p+(1-p)*F(gtau(p))-x,.5);
```

Empirical Implementation

- Once you have an estimate for p, denoted $\hat{p}$
- You can quantify how often do managers strategically withhold information

$$\Pr\left(\text{strategic withholding}\right) = \left(1 - \hat{p}\right)\Phi\left(\tau\left(\hat{p}\right)\right)$$

Title Text

Table 4.

Prevalence and Impact of Strategic Withholding: Static Model

This table reports estimates of equations (8), (9), and (10). ξ is the probability of strategic withholding, $\mathbb{E}(Var(\frac{e_t}{\sigma_e}|d_t x_t))$ is the average residual variance of earnings as defined in Dye and Hughes (2018),$\mathbb{E}^{ns}(Var(\frac{e_t}{\sigma_e}|d_t x_t))$ is the non-strategic version of the average residual variance of earnings, and $\mathbb{E}(\frac{e_t}{\sigma_e}|d_t = 0)$ is the overvaluation of earnings incurred by a naïve investor given non-disclosure. Bootstrapped standard errors appear in parentheses below each estimate, using the sample construction in Table 1.

ξ	$\mathbb{E}(Var(\frac{e_t}{\sigma_e}\|d_t x_t))$	$\mathbb{E}^{ns}(Var(\frac{e_t}{\sigma_e}\|d_t x_t))$	$\mathbb{E}(\frac{e_t}{\sigma_e}\|d_t = 0)$
0.2186	0.8954	0.8541	-0.2495
(.0015)	(.0137)	(.0189)	(.0181)

Disclosure is Persistent

- If the previous game was repeated every period, then disclosure decisions d_t would be iid
- In reality, when one looks at the time series of disclosure choices, there is a lot of serial correlation
- That is, it's more likely that a firm discloses in t if it has done it in the $t-1$, and vice-versa
- To address this issue, one needs a dynamic model, that allows for forward looking considerations and creates persistence of disclosure decisions

Subsample Moments:	**data (se)**
$E(d_t = 1 \mid \text{br.})$	.47 (.01)
$P(d_t = 1 \mid d_{t-1} = 1, \text{br.})$	.82 (.01)
$P(d_t = 1 \mid d_{t-2} = 1, \text{br.})$	.74 (.01)
$P(d_t = 1 \mid d_{t-3} = 1, \text{br.})$	.68 (.01)
Std $(s_t - e_t \mid d_t = 1, \text{br.})$	.01 (.00)
$P(\ d_t = 1$ Never)	.39 (.01)
$P(d_t = 1$ Always)	.16 (.01)
$\text{Corr}(e_t,e_{t-1})$	.39 (.02)
Std (e_t)	.04 (.00)
Std $(c_t - e_t)$	.03 (.00)

A Dynamic Model

- The following is a dynamic extension of Dye (1985) with forward-looking motives (see Einhorn and Ziv 2008).
- Time is indexed by $t = 0, 1, 2, ...$ and, in each period t, the manager maximizes

$$\mathcal{U}_t = \mathbb{E}(\sum_{n=t}^{\infty} \beta^{n-t} P_n | \mathcal{I}_t)$$

  where β is the manager's discount factor, P_n is the market price and $\mathcal{I}_t$ is the manager's information set.

A Dynamic Model

The timeline is as follows.

1. In each period the manager may or may not have information about x_t. The manager's information endowment is a random variable $\theta_t \in \{0, 1\}$ where $\theta_t = 1$ represents the case when the manager is informed.
2. This random variable is a Markov chain such that $\Pr(\theta_t = 0|\theta_{t-1} = 0) = k_0$ and $\Pr(\theta_t = 0|\theta_{t-1} = 1) = k_1$, with $k_0, k_1 \in (0, 1)$. You can simulate this Markov chain in Matlab.

```
T=10
k0=.5;
k1=.2;
sigma_x=1
P=[k0 1-k0;k1 1-k1];
mc=dtmc(P);
figure;
graphplot(mc,'ColorEdges',true)
s=simulate(mc,T,'X0',[0 1]);
x=sigma_x*randn(T,1);
theta=s-1;
```

A Dynamic Model

1. The state θ_t is observed by the manager alone. When $\theta_t = 1$, the manager observes a signal x_t, where $x_t \sim N(0, \sigma_x^2)$.
2. The manager may voluntarily disclose x_t or conceal it. We use the convention that $d_t = x_t$ when x_t is voluntarily disclosed and $d_t = ND$ otherwise.
3. The market observes the disclosure, if any, and then rationally prices the current earnings at $P_t = \mathbb{E}(x_t | \mathcal{I}_t^0, d_t)$, where $\mathcal{I}_t$ represents the public information set.

The public information set $\mathcal{I}_t$ is then updated to $\mathcal{I}_t^0 = \{\mathcal{I}_{t-1}^0, d_t\}$.

Caveats

We are making two strong assumptions:

1. x_t is iid, and
2. The only source of information is the manager forecast. In particular, if the manager does not disclose x_t, there is no public information coming out, such as a mandatory earnings announcement, that would reveal x_t later on.
3. These assumptions can easily be relaxed (See BMTV 2020)

Solving the model

- If $k_0 = k_1$ the information endowment is iid and we have a repeated version of the static model.
- In general, the manager has forward looking motives, because a disclosure today affects the market's future beliefs about the information endowment.
- In particular if $k_0 > k_1$ the manager is more likely to remain uniformed if he starts from the uninformed state. By not disclosing the manager may try to convince the market he is uninformed today hence likely to be uninformed in the future.

Solving the model

- Focus on Markov Perfect Equilibria
- State variable p_t representing the belief of the market that the manager is informed in period t
- These beliefs need to be consistent with Bayes' rule in equilibrium

Solving the model

- Fix the market belief at the start of a period, p_t
- Conjecture an equilibrium in which the manager withholds his information, if any, when his signal is such that $x_t < \tau(p_t)$ where the threshold function $\tau(\cdot)$ depends on p_t.
- We need to solve for the equilibrium $\tau(\cdot)$
- If the market anticipates this strategy, then the price of non disclosure is computed as

$$\begin{aligned} P^{ND}(p_t) &= \frac{(1-p_t)\,\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right) E[x_t | x_t \leq \tau(p_t)] + p_t E(x_t)}{p_t + (1-p_t)\,\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)} \\ &= -\frac{\sigma_x (1-p_t)\,\phi\left(\frac{\tau(p_t)}{\sigma_x}\right)}{p_t + (1-p_t)\,\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)}. \end{aligned} \tag{3}$$

- where we have used the fact that $E(x_t) = 0$

Solving the model

- Also, given no disclosure, the market updates its belief using Bayes' rule as follows
- Then the next period belief conditional on no disclosure is given by

$$p_{t+1} = \Pr\left(\theta_t = 0 \middle| d_t = ND\right) \cdot k_0 + \Pr\left(\theta_t = 1 \middle| d_t = ND\right) \cdot k_1 \qquad (4)$$

where

$$\Pr\left(\theta_t = 0 \middle| d_t = ND\right) = \frac{p_t}{p_t + (1 - p_t)\,\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)}.$$

- Of course, given a disclosure investors know that $\theta_t = 1$ hence $p_{t+1} = k_1$.

The value functions

- Denote $v_1(x_t, p_t)$ the equilibrium payoff to a manager who is informed $\theta_t = 1$ when the signal is x_t and the market assigns probability p_t to the manager being uninformed.
- Let $v_0\left(p_t\right)$ the equilibrium payoff to a manager who is uninformed $\theta_t = 0$. Then

$$v_0(p_t) = P_{ND}\left(p_t\right) + \beta\mathbb{E}(k_0 \cdot v_0(p_{t+1}) + (1 - k_0) \cdot v_1(x_{t+1}, p_{t+1})).$$

  where p_{t+1} is given by equation (4).
- Now define $U^{ND}\left(p_t\right)$ as the payoff to an informed manager who chooses to not disclose his information:

$$U^{ND}\left(p_t\right) = P_{ND}\left(p_t\right) + \beta\mathbb{E}(k_1 \cdot v_0(p_{t+1}) + (1 - k_1) \cdot v_1(x_{t+1}, p_{t+1})). \tag{5}$$

Solving the model

- By contrast, if an informed manager discloses his signal, $d_t = x_t$, then in equilibrium his payoff must be

$$x_t + \underbrace{\beta \mathbb{E}\left[\sum_{k=t+1}^{\infty} \beta^{k-(t+1)} x_k\right]}_{=0} = x_t$$

- Now $\tau(p_t)$ is an equilibrium disclosure threshold, if a manager observing $x_t = \tau(p_t)$ is indifferent between disclosing and not disclosing, hence

$$\tau(p_t) = U^{ND}(p_t)$$

- More generally we have

$$v_1(x_t, p_t) = \max\left(x_t, U^{ND}(p_t)\right).$$

Solving the model

- We can integrate x_t out and define

$$
\begin{aligned}
v_1(p_t) &\equiv \mathbb{E}\left[\max\left(x_t, U^{ND}(p_t)\right)\right] && (6)\\
&= \mathbb{E}\left[\max\left(x_t, \tau(p_t)\right)\right] && (7)\\
&= \Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)\tau(p) + \sigma_x \phi\left(\frac{\tau(p_t)}{\sigma_x}\right)
\end{aligned}
$$

Solving the model

- Then we can re-write equation (5) as

$$U^{ND}(p_t) = P_{ND}(p_t) + \beta(k_1 \cdot v_0(p_{t+1}) + (1-k_1) \cdot v_1(p_{t+1})). \quad (8)$$

- Notice that the market breaks even in equilibrium (no real effects) hence

$$p_{t+1} v_0(p_{t+1}) + (1-p_{t+1}) v_1(p_{t+1}) = E\left[\sum_{k=t+1}^{\infty} \beta^{k-(t+1)} x_k\right] = 0.$$

- which implies that $v_0(p_{t+1}) = -\frac{(1-p_{t+1})}{p_{t+1}} v_1(p_{t+1})$. Plugging this expression into equation 8, yields the following equation for $\tau(\cdot)$

$$\tau(p_t) = P_{ND}(p_t) + \beta(1 - k_1 - k_1 \frac{1-p_{t+1}}{p_{t+1}}) \cdot v_1(p_{t+1}) \quad (9)$$

- subject to equations (4) and (6) and (3).

Dynamic Model

- To solve this model, one can iterate equation (9) over a grid of p values to obtain $v_1(p)$ and $\tau(p)$ and everything else.

```
function [X tau]=vfi(par)

k0=.5;
k1=.2;
beta=.75;
N=100;
gridsize=N;
X=linspace(1/gridsize,1-1/gridsize,gridsize);
f=@(t) normpdf(t);
F=@(t) normcdf(t);

%Price of non disclosure formula

pnd=@(t,p) (-(1-p)*f(t))/(p+(1-p)*F(t));


%Initialize value function
```

```
V=@(q) 0;

% belief updating and net utility functions
p2=@(t,p) p/(p+(1-p)*F(t))*k0+(1-p/(p+(1-p)*F(t)))*k1;

%Utility of non disclosure

Und=@(t,p) pnd(t,p)...

+beta*((1-k1)-k1*(1-p2(t,p))/p2(t,p))*V(p2(t,p));

%Value function iteration

stopcond=1;
while stopcond>0.1
    for i=1:gridsize
    p0=X(i);
    [t0(i)]=fminsearch(@(t) (Und(t,p0)-t)^2,0);
    V2(i)=quadgk(@(x) max(x,Und(t0(i),p0)).*f(x),-3,3);
```

```
    end

V_new=@(p) interp1(X,V2,p);
stopcond=max(V_new(X)-V(X));
V=V_new;
end

tau=t0;
```

Estimating the Model

- In this model, disclosure decisions are forward looking.
- We can estimate this model by Maximum Likelihood.
- The log-likelihood of a sequence of disclosures is $\mathcal{L}(\zeta) = \sum_{t=1}^{T} \ln f_t(d_t|\zeta)$.
  where $\zeta = \{k_0, k_1, \sigma_x, \beta\}$ and the conditional pdf of d_t given the history and parameters is

$$f_t(d_t|\zeta) = \begin{cases} (1-p_t)\frac{1}{\sigma_x}\phi\left(\frac{x_t}{\sigma_x}\right) & d_t = x_t, d_t \geq \tau(p_t) \\ 0 & d_t = x_t, d_t < \tau(p_t) \\ p_t + (1-p_t)\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right) & d_t = ND \end{cases}.$$

- and recall that the law of the market beliefs follows

$$p_{t+1} = \begin{cases} k_1 & \text{if} \quad d_t = x_t \\ \frac{p_t}{p_t+(1-p_t)\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)} \cdot k_0 + \frac{(1-p_t)\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)}{p_t+(1-p_t)\Phi\left(\frac{\tau(p_t)}{\sigma_x}\right)} \cdot k_1 & \text{if} \quad d_t = ND \end{cases},$$

Estimating the Model

- What about p_1? We assume that $p_1 = k_1$ as if there was disclosure in period $t = 0$.
- The MLE estimator is defined as

$$\hat{\zeta}^{MLE} = \arg\max \mathcal{L}(\zeta)$$

- The asymptotic distribution is

$$\hat{\zeta}^{MLE} \sim N(\zeta, \mathbf{I}(\zeta)^{-1}),$$

- where the information matrix $\mathbf{I}(\zeta)$ can be estimated in different ways. For example,

$$\hat{\mathbf{I}} = \frac{-\sum_{t=1}^{T} \mathbf{H}\left(d_t, \hat{\zeta}\right)}{T}.$$

Billionaire investor Warren Buffett says he is fine with companies releasing their results on a quarterly basis, but he can do without their quarterly forecasts.

"I like to read quarterly reports as an investor," Buffett, the chairman and CEO of Berkshire Hathaway, told CNBC's Becky Quick on Thursday. "I like to get those quarterly reports. I do not like guidance. I think the guidance leads to a lot of bad things, and I've seen it lead to a lot of bad things."

Buffett's comments come nearly three months after he and J.P. Morgan Chase CEO Jamie Dimon told CNBC they wanted CEOs to stop issuing quarterly profit forecasts.

On CNBC Thursday, he said, "I think it's a very bad practice to be in the game of earnings guidance, and it is a game."

Conclusions

- This model offers an opportunity to estimate manager's myopia β via disclosure choices
- This model can be extended to include investment decisions to measure the impact of strategic disclosure on real decisions
- Another option is to add the possibility of misreporting
- Consider the effect of investor risk aversion on disclosure choices
- Even simple models can be powerful empirical tools
- Results have limitations: be aware of them and forthcoming